

MATH MILESTONE # B3

FACTORS

The word, **milestone**, means “a point at which a significant change occurs.” A Math Milestone refers to a significant point in the understanding of mathematics.

To reach this milestone one should be able to use factors in simplifying math computations.

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A calculator shall be required to check the answers arrived at through mental math.

Please consult the [Glossary](#) supplied with this Milestone for mathematical terms. Consult a regular dictionary at www.dictionary.com for general English words that one does not understand fully.

You may start with the Diagnostic Test on the next page to assess your proficiency on this milestone. Then continue with the lessons with special attention to those, which address the weak areas.

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DIAGNOSTIC TEST

1. Determine the following.

(a) Is 4 a factor of 13?	(c) Is 7 a factor of 49?	(e) Is 6 a factor of 56?
(b) Is 13 a factor of 91?	(d) Is 9 a factor of 72?	(f) Is 8 a factor of 42?
2. Write a pair of factors for the following numbers (there is more than one answer).

(a) 18 = ___ x ___	(c) 32 = ___ x ___	(e) 40 = ___ x ___
(b) 33 = ___ x ___	(d) 23 = ___ x ___	(f) 75 = ___ x ___
3. Identify the PRIME numbers.
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 21, 23, 31, 39, 41, 49, 53, 57, 67, 79, 87, 91
4. Rapidly identify the numbers divisible by 5.
32, 65, 44, 87, 40, 57, 78, 100, 174, 465, 999, 5390, 8883, 5555, 9999993
5. Rapidly identify numbers divisible by 3.
32, 63, 44, 87, 66, 57, 78, 100, 174, 465, 999, 5392, 8883, 5556, 9999993
6. Write all even prime numbers less than 100.
7. Write all prime numbers between 100 and 120.
8. Find the prime factors of the following numbers.
36, 88, 630, 1155, 2401
9. Divide by taking out the common factors

(a) $36 \div 12$	(c) $806 \div 26$	(e) $7920 \div 240$
(b) $270 \div 18$	(d) $544 \div 32$	(f) $17640 \div 630$
10. Determine the Greatest Common Factor (GCF) of.

(a) 18 and 27	(c) 54 and 258	(e) 162, 729 and 4374
(b) 30 and 42	(d) 216 and 264	(f) 492, 744 and 1044

ANSWER: 1. (a) No (b) Yes (c) Yes (d) Yes (e) No (f) No 2. (a) 3×6 (b) 3×11 (c) 8×4 (d) 23×1 (e) 4×10
 (f) 3×25 3. 2, 3, 5, 7, 11, 23, 31, 41, 53, 67, 79 4. 65, 78, 100, 465, 999, 5390, 8883, 5556, 9999993 5. 6, 32, 63, 78, 100, 174, 465, 999, 5392, 8883, 5556, 9999993
 6. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98
 7. 101, 103, 107, 109, 113, 127 8. $3 \times 3 \times 2 \times 2 \times 3$, $2 \times 2 \times 2 \times 3 \times 3 \times 5$, $2 \times 2 \times 3 \times 3 \times 5 \times 7$, $3 \times 3 \times 5 \times 7 \times 11$, $2401 = 7 \times 7 \times 7 \times 7$ 9. (a) 3 (b) 15 (c) 31
 (d) 17 (e) 33 (f) 28 10. (a) 9 (b) 6 (c) 6 (d) 24 (e) 81 (f) 12

LESSONS

Lesson B3.1 Exact Division and Factors

A FACTOR is an exact divisor of a number.

- (a) 4 is an exact divisor of 24. Therefore, 4 is a factor of 24
(b) 7 is an exact divisor of 91. Therefore, 7 is a factor of 91

$$91 \div 7 = 13 \quad \text{(Exact Division)}$$

↑
Factor

- In exact division, the divisor and quotient may be interchanged. Therefore, the quotient is also a factor of the dividend,

$$91 \div 7 = 13 \quad \rightarrow \quad 91 \div 13 = 7 \quad \rightarrow \quad \text{both 7 and 13 are factors of 91}$$

$$91 \div 13 = 7 \quad \text{(Exact Division)}$$

↑
Factor

- Division is the reverse of multiplication. The divisor and quotient become, in multiplication, the factors of the product.

$$\text{If } 91 \div 7 = 13, \quad \text{then } 13 \times 7 = 91 \quad \rightarrow \quad \text{both 13 and 7 are factors of 91}$$

$$13 \times 7 = 91$$

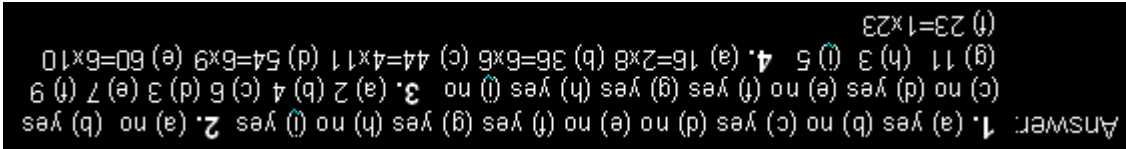
↑ ↑ ↑
Factor Factor Product

☺ Exercise B3.1

- State if the divisor is a factor of the dividend.
(a) $9 \div 3$ (d) $17 \div 5$ (g) $20 \div 4$
(b) $9 \div 2$ (e) $21 \div 4$ (h) $20 \div 3$
(c) $6 \div 2$ (f) $28 \div 7$ (i) $12 \div 4$
- Answer the following
(a) Is 4 a factor of 13? (d) Is 7 a factor of 49? (g) Is 6 a factor of 48?
(b) Is 7 a factor of 35? (e) Is 6 a factor of 56? (h) Is 13 a factor of 91?
(c) Is 5 a factor of 28? (f) Is 8 a factor of 56? (i) Is 15 a factor of 130?
- Find the missing factor.
(a) $4 = 2 \times \underline{\quad}$ (d) $6 = 2 \times \underline{\quad}$ (g) $22 = 2 \times \underline{\quad}$
(b) $16 = 4 \times \underline{\quad}$ (e) $21 = 3 \times \underline{\quad}$ (h) $15 = 5 \times \underline{\quad}$
(c) $24 = 4 \times \underline{\quad}$ (f) $27 = 3 \times \underline{\quad}$ (i) $25 = 5 \times \underline{\quad}$

4. Write a pair of factors for the following numbers.

- (a) 16 = ____ x ____ (d) 54 = ____ x ____
 (b) 36 = ____ x ____ (e) 60 = ____ x ____
 (c) 44 = ____ x ____ (f) 23 = ____ x ____



Lesson B3.2 Prime & Composite Numbers

PRIME numbers have only two different factors (1 and itself). COMPOSITE numbers have more than two different factors.

1. All number may be expressed as the product of 1 and itself.

- $4 = 1 \times 4 \rightarrow$ 1 and 4 are factors of 4
 $17 = 1 \times 17 \rightarrow$ 1 and 17 are factors of 17
 $20 = 1 \times 20 \rightarrow$ 1 and 20 are factors of 20
 $23 = 1 \times 23 \rightarrow$ 1 and 23 are factors of 23

2. A Prime number does not have factors other than 1 and itself.

- $17 = 1 \times 17 \rightarrow$ 1 and 17 are the only factors of 17
 \rightarrow 17 is a prime number
 $23 = 1 \times 23 \rightarrow$ 1 and 23 are the only factors of 23
 \rightarrow 23 is a prime number

3. A COMPOSITE number has factors other than 1 and itself.

- $4 = 2 \times 2 \rightarrow$ 4 has 2 as a factor other than 1 and 4
 \rightarrow 4 is a composite number
 $20 = 4 \times 5 \rightarrow$ 20 has 4 and 5 as factors other than 1 and 20
 \rightarrow 20 is a composite number

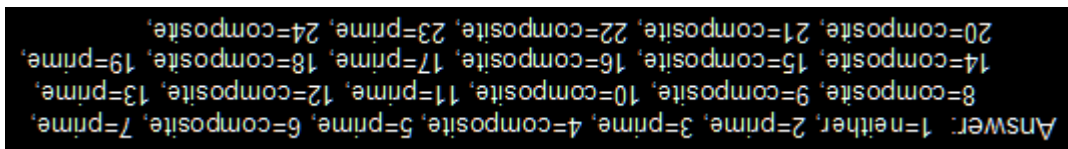
4. 1 is neither composite nor prime because it has only one factor.

- $1 = 1 \times 1 \rightarrow$ 1 is not a prime number because it does not have two different factors.
 $1 = 1 \times 1 \rightarrow$ 1 is not a composite number because it does not have more than two different factors.

☺ Exercise B3.2

1. Identify the following numbers as PRIME or COMPOSITE.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24.



Lesson B3.3 Determining Prime Numbers

Prime numbers may be determined as follows.

1. The single-digit prime numbers are 2, 3, 5, and 7.

(a) 1 is neither composite nor prime as explained earlier.

$$1 = 1 \times 1 \quad \rightarrow \quad 1 \text{ is not a prime number}$$

(b) EVEN NUMBERS are those for which all units may be paired up. All even numbers are divisible by 2. Even numbers, other than 2, are not prime numbers.

$$2 = 2 \times 1 \quad \rightarrow \quad 2 \text{ is a prime number}$$

$$4 = 2 \times 2 \quad \rightarrow \quad 4 \text{ is not a prime number}$$

$$6 = 2 \times 3 \quad \rightarrow \quad 6 \text{ is not a prime number}$$

$$8 = 2 \times 4 \quad \rightarrow \quad 8 \text{ is not a prime number}$$

(c) ODD NUMBERS are those for which, upon pairing, one unit is left unpaired. All prime numbers, other than 2, are odd numbers. But not all odd numbers are prime numbers.

$$3 = 3 \times 1 \quad \rightarrow \quad 3 \text{ is a prime number}$$

$$5 = 5 \times 1 \quad \rightarrow \quad 5 \text{ is a prime number}$$

$$7 = 7 \times 1 \quad \rightarrow \quad 7 \text{ is a prime number}$$

$$9 = 3 \times 3 \quad \rightarrow \quad 9 \text{ is not a prime number}$$

The single-digit PRIME NUMBERS are: **2, 3, 5, and 7.**

2. The double-digit prime numbers are those that cannot be divided by single-digit prime numbers.

(a) Here we have all double-digit numbers.

10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

(b) Remove all numbers that are exactly divisible by 2, that is, remove all even numbers.

*	11	*	13	*	15	*	17	*	19
*	21	*	23	*	25	*	27	*	29
*	31	*	33	*	35	*	37	*	39
*	41	*	43	*	45	*	47	*	49
*	51	*	53	*	55	*	57	*	59
*	61	*	63	*	65	*	67	*	69
*	71	*	73	*	75	*	77	*	79
*	81	*	83	*	85	*	87	*	89
*	91	*	93	*	95	*	97	*	99

(c) Remove all numbers that are exactly divisible by 5.

*	11	*	13	*	*	*	17	*	19
*	21	*	23	*	*	*	27	*	29
*	31	*	33	*	*	*	37	*	39
*	41	*	43	*	*	*	47	*	49
*	51	*	53	*	*	*	57	*	59
*	61	*	63	*	*	*	67	*	69
*	71	*	73	*	*	*	77	*	79
*	81	*	83	*	*	*	87	*	89
*	91	*	93	*	*	*	97	*	99

(d) Remove all numbers that are exactly divisible by 3.

*	11	*	13	*	*	*	17	*	19
*	*	*	23	*	*	*	*	*	29
*	31	*	*	*	*	*	37	*	*
*	41	*	43	*	*	*	47	*	49
*	*	*	53	*	*	*	*	*	59
*	61	*	*	*	*	*	67	*	*
*	71	*	73	*	*	*	77	*	79
*	*	*	83	*	*	*	*	*	89
*	91	*	*	*	*	*	97	*	*

(e) Remove all numbers that are exactly divisible by 7.

*	11	*	13	*	*	*	17	*	19
*	*	*	23	*	*	*	*	*	29
*	31	*	*	*	*	*	37	*	*
*	41	*	43	*	*	*	47	*	*
*	*	*	53	*	*	*	*	*	59
*	61	*	*	*	*	*	67	*	*
*	71	*	73	*	*	*	*	*	79
*	*	*	83	*	*	*	*	*	89
*	*	*	*	*	*	*	97	*	*

The remaining double-digit numbers are PRIME NUMBERS:

11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97

3. To find prime numbers beyond 100, simply divide the number by successive prime numbers starting from 2 until there is no remainder, or the quotient becomes less than the divisor.

(a) Determine if **143** is a prime number.

$143 \div 2$	\rightarrow	71 R1	(2 is not a factor)
$143 \div 3$	\rightarrow	47 R2	(3 is not a factor)
$143 \div 5$	\rightarrow	28 R3	(5 is not a factor)
$143 \div 7$	\rightarrow	20 R3	(7 is not a factor)
$143 \div 11$	\rightarrow	13	(no remainder means 11 is a factor)
	\rightarrow	143	is not a prime number

(b) Determine if **149** is a prime number.

- $149 \div 2 \rightarrow 74 \text{ R}1$ (2 is not a factor)
 $149 \div 3 \rightarrow 49 \text{ R}2$ (3 is not a factor)
 $149 \div 5 \rightarrow 29 \text{ R}4$ (5 is not a factor)
 $149 \div 7 \rightarrow 21 \text{ R}2$ (7 is not a factor)
 $149 \div 11 \rightarrow 13 \text{ R}6$ (11 is not a factor)
 $149 \div 13 \rightarrow 11 \text{ R}6$ (13 is not a factor)
 \rightarrow The quotient 11 is less than the divisor 13, so we stop
 \rightarrow No factors are found, therefore, **149** is a prime number

(c) The following is a table of prime numbers up to the beginning of 4-digit prime numbers.

PRIME NUMBERS

2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	1009
59	139	233	337	439	557	653	769	883	1013

😊 Exercise B3.3

- Write all even prime numbers less than 10.
- Write all odd prime numbers less than 10.
- Identify the prime numbers among the following by dividing them by 2, 3, 5, and 7.
21, 23, 28, 31, 33, 37, 39, 41, 43, 49, 51, 53, 57, 67, 74, 79, 81, 87, 91, 97
- Find the next ten prime numbers after 1013 after completing this milestone.

Answer: 1. The only even prime number is 2. 2. Odd prime numbers less than 10 are 3, 5, and 7. 3. 23, 31, 37, 41, 43, 53, 59, 67, 79, and 97.

Lesson B3.4 Divisibility of Numbers

The divisibility of numbers may be determined as follows.

1. 2 is a factor of a number whose last digit can be divided by 2.
 - (a) The last digit of 54 is 4 → 4 is divisible by 2 → 2 is a factor of 54.
The last digit of 73 is 3 → 3 is not divisible by 2 → 2 is not a factor of 73.
The last digit of 436 is 6 → 6 is divisible by 2 → 2 is a factor of 436.
 - (b) 2 is a factor of all numbers ending with 2, 4, 6, 8 or 0.
2, 4, 6, 8, and 10 are divisible by 2. Therefore, all numbers ending with 2, 4, 6, 8, and 0 are also divisible by 2.
2. 3 is a factor of a number whose digits add up to a sum that can be divided by 3.
 - (a) We add the digits of the number to find out if their sum is divisible by 3
The digits of **54** add up to 9 → 9 is divisible by 3 → **3** is a factor of **54**
The digits of **251** add up to 8 → 8 is not divisible by 3 → **3** is not a factor of **251**
The digits of **321** add up to 6 → 6 is divisible by 3 → **3** is a factor of **321**
 - (b) 3 is a factor of all numbers whose digits ultimately add up to 3, 6, or 9.
The digits of **897** add up to 24
→ the digits of 24 add up to 6
→ **3** is a factor of **897**
The digits of **18,796** add up to 31
→ the digits of 31 add up to 5
→ **3** is not a factor of **18,796**
The digits of **78,916,545** add up to 45
→ the digits of 45 add up to 9
→ **3** is a factor of **78,916,545**
3. 4 is a factor of a number whose last 2 digits can be divided by 4.
 - (a) 2 is a factor of 4. Therefore, 2 is also a factor for the number of which 4 is a factor. If a number is odd then 4 cannot be a factor of that number.
 - (b) We isolate the last two digits as a number, and check if it is divisible by 4.
Isolate the last two digits of **612** as 12
→ 12 is divisible by 4
→ **4** is a factor of **612**
Isolate the last two digits of **746** as 46
→ 46 is not divisible by 4
→ **4** is not a factor of **746**
Isolate the last two digits of **45,308** as 08 or 8
→ 8 is divisible by 4
→ **4** is a factor of **45,308**

4. 5 is a factor of a number whose last digit is 0 or 5. We isolate the last digit of a number, and check if it is '0' or '5'.

The last digit of **735** is 5 → **5** is a factor of **735**
 The last digit of **981** is 1 → **5** is not a factor of **581**
 The last digit of **33,330** is 0 → **5** is a factor of **33,330**

5. 6 is a factor of a number that has 2 and 3 as its factors. We check if the number is even, and also if it is divisible by 3.

The number **654** is even → It is also divisible by 3 → **6** is a factor of **654**
 The number **453** is not even → **6** is not a factor of **453**
 The number **772** is even → It is not divisible by 3 → **6** is a not factor of **772**
 The number **10,008** is even → It is also divisible by 3 → **6** is a factor of **10,008**

6. 7 as a factor may be determined by the following procedure.

- (a) Take the digits from left up to TENS of a number. Subtract the double of ONES. If the remainder is divisible by 7, then the original number is also divisible by 7.

Consider the number **749**.

Digits up to TENS = 74
 Double of ONES = $9 * 2 = 18$
 Difference = $74 - 18 = 56$

Since 56 is divisible by 7, the number 749 is also divisible by 7.

- (b) For a large number, you may repeat this process till you arrive with a two digit number.

Consider the number **38073**.

Digits up to TENS = 3807
 Double of ONES = $3 * 2 = 6$
 Difference = $3807 - 6 = 3801$

Repeat the process with 3801

Digits up to TENS = 380
 Double of ONES = $1 * 2 = 2$
 Difference = $380 - 2 = 378$

Repeat the process with 378

Digits up to TENS = 37
 Double of ONES = $8 * 2 = 16$
 Difference = $37 - 16 = 21$

Since 21 is divisible by 7, 38073 is also divisible by 7.

- (c) With a little practice, you will be able to do this at lightning speed!

Consider the number **45717**.

4571 - 14 = 4557 → **455 - 14 = 441** → **44 - 2 = 42**
 42 is divisible by 7. Therefore, **45717** is also divisible by 7.

7. 8 is a factor of a number that is even and whose last 3 digits can be divided by 8.

- (a) For a number to be divisible by 8, it should also be divisible by 2 and 4.
 (b) We isolate the last three digits as a number, and check if it is divisible by 8.

The last three digits of **1,016** are '016'
 → 16 is divisible by 8
 → **8 is a factor of 1,016**

The last three digits of **32,483** are '483'
 → 483 is an odd number (not divisible by 2 or 8)
 → **8 is not a factor of 32,483**

The last three digits of **5,814** are '814'
 → '14' is not divisible by 4
 → **8 is not a factor of 5,814**

The last three digits of **77,416** are '416'
 → 416 is divisible by 2, 4, and 8
 → **8 is a factor of 77,416**

8. 9 is a factor of a number whose digits add up to a sum that can be divided by 9 .

(a) For a number to be divisible by 9, it should also be divisible by 3.

(b) We add the digits of the number to find out if their sum is divisible by 9

The digits of **54** add up to 9 → 9 is divisible by 9 → **9 is a factor of 54**

The digits of **251** add up to 8 → 8 is not divisible by 9 → **9 is not a factor of 251**

The digits of **321** add up to 6 → 6 is not divisible by 9 → **9 is not a factor of 321**

The digits of **39,321** add up to 18 → 18 is divisible by 9 → **9 is a factor of 39,321**

(c) 9 is a factor of all numbers whose digits ultimately add up to 9.

The digits of **8,973** add up to 27
 → the digits of 27 add up to 9
 → **9 is a factor of 8,973**

The digits of **18,796** add up to 31
 → the digits of 31 add up to 4
 → **9 is not a factor of 18,796**

The digits of **78,916,545** add up to 45
 → the digits of 45 add up to 9
 → **9 is a factor of 78,916,545**

9. 10 is a factor of all numbers, whose last digit is 0. We isolate the last digit of a number, and check if it is '0'.

The last digit of **100** is 0 → **10 is a factor of 100**

The last digit of **765** is 5 → **10 is not a factor of 765**

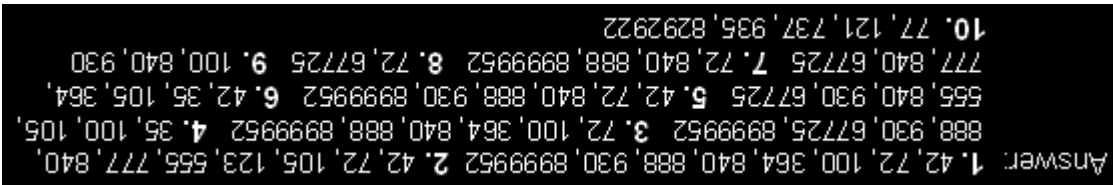
The last digit of **33,330** is 0 → **10 is a factor of 33,330**

10. 11 is a factor of a number when the difference between the sum of digits in odd places and the sum of digits in even places is 0, or can be divided by 11.

(a) Get the sum of digits in odd places, and then get the sum of digits in even places. Find the difference between the two sums, and check if it is 0 or a multiple of 11.

☺ **Practice #1** (Note: Commas are not used within the numbers below.)

1. Circle the following numbers for which 2 is a factor.
42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
2. Circle the following numbers for which 3 is a factor.
42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
3. Circle the following numbers for which 4 is a factor.
42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
4. Circle the following numbers for which 5 is a factor.
42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
5. Circle the following numbers for which 6 is a factor.
42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
6. Circle the following numbers for which 7 is a factor.
42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
7. Circle the following numbers for which 8 is a factor.
42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
8. Circle the following numbers for which 9 is a factor.
42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
9. Circle the following numbers for which 10 is a factor.
42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
10. Circle the following numbers for which 11 is a factor.
42, 35, 77, 100, 111, 121, 364, 555, 737, 840, 888, 935, 67725, 8292922

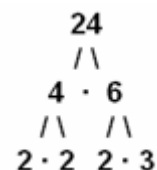


Lesson B3.5 Factoring of Numbers

*When a number is factored again and again until it cannot be factored any further, then we have **PRIME FACTORS**.*

1. We factor a number till it cannot be factored any further

(a) This may be pictured as a **FACTOR TREE** as follows. Note: The operation of multiplication may be shown by a dot (.).



(b) Here the factors 4 and 6 are factored further. The factors 2 and 3 cannot be factored any further so we stop. Thus, 24 may be shown in terms of prime factors as follows.

$$24 = 4 \times 6 = (2 \times 2) \times (2 \times 3) = 2 \times 2 \times 2 \times 3$$

(c) Similarly, we may show 60 in terms of its prime factors as follows.

$$60 = 4 \times 15 = (2 \times 2) \times (3 \times 5) = 2 \times 2 \times 3 \times 5$$

2. The set of prime factors is unique to a number. No two numbers can have the same set of prime factors.

"2, 3 and 7" are a unique set of prime factors for 42 because

$$2 \times 3 \times 7 = 42$$

"2, 2, 5 and 5" are a unique set of prime factors for 100 because

$$2 \times 2 \times 5 \times 5 = 100$$

"2, 3, 5 and 7" are a unique set of prime factors for 210 because

$$2 \times 3 \times 5 \times 7 = 210$$

3. The prime factors of a number may be found by dividing the number by successive prime numbers beginning with 2. *Note: The quotient is shown below the number.*

- (a) Find the prime factors of **24** using short division.

We check the divisibility by successive prime numbers, and then divide.

Each time we check starting from the last divisor. The prime factors are the divisors and the final quotient.

$$24 = 2 \times 2 \times 2 \times 2 \times 3$$

$$\begin{array}{r} 2 \overline{) 24} \\ \underline{2} \\ 2 \\ \underline{2} \\ 0 \\ 2 \overline{) 6} \\ \underline{6} \\ 0 \end{array}$$

- (b) Find the prime factors of **35574**.

One divides by successive prime numbers until the quotient is also a prime number.

$$35574 = 2 \cdot 3 \cdot 7 \cdot 7 \cdot 11 \cdot 11$$

$$\begin{array}{r} 2 \overline{) 35574} \\ \underline{2} \\ 3 \overline{) 17787} \\ \underline{3} \\ 7 \overline{) 5929} \\ \underline{7} \\ 7 \overline{) 847} \\ \underline{7} \\ 11 \overline{) 121} \\ \underline{11} \\ 0 \end{array}$$

- (c) Find the prime factors of **2310**.

$$\begin{aligned} 2310 &= 10 \times 231 \\ &= (2 \times 5) \times (3 \times 7 \times 11) \\ &= 2 \times 3 \times 5 \times 7 \times 11 \end{aligned}$$

$$2 \overline{) 10} \\ \underline{2} \\ 5$$

$$\begin{array}{r} 3 \overline{) 231} \\ \underline{3} \\ 7 \overline{) 77} \\ \underline{7} \\ 11 \end{array}$$

- (d) Find the prime factors of **71996**.

Since we have been successively checking the prime numbers, 439 does not have any factors up to 41.

$$439 \div 41 = 10 \text{ R}29$$

439 is not divisible by 41, and the resulting quotient is less than 41.

So we stop here.

$$71996 = 2 \times 2 \times 41 \times 439$$

$$\begin{array}{r} 2 \overline{) 71996} \\ \underline{2} \\ 2 \overline{) 35998} \\ \underline{2} \\ 41 \overline{) 17999} \\ \underline{41} \\ 439 \end{array}$$

☺ Exercise B3.5

1. Show the following numbers in terms of their prime factors.

9, 15, 18, 27, 36, 39, 49, 58, 64, 75, 88, 93, 100

2. The following are a unique set of prime factors of what number?

- | | | |
|------------------------------------|---------------------------|---------------------------|
| (a) $2 \times 3 \times 3$ | (c) $2 \times 3 \times 5$ | (e) $3 \times 5 \times 7$ |
| (b) $2 \times 2 \times 2 \times 3$ | (d) $3 \times 5 \times 5$ | (f) $3 \times 3 \times 5$ |

Answer: 1. $9=3 \times 3$, $15=3 \times 5$, $18=2 \times 3 \times 3$, $27=3 \times 3 \times 3$, $36=2 \times 2 \times 3 \times 3$, $39=3 \times 13$, $49=7 \times 7$, $58=2 \times 29$, $64=2 \times 2 \times 2 \times 2 \times 2$, $75=3 \times 5 \times 5$, $88=2 \times 2 \times 2 \times 11$, $93=3 \times 31$, $100=2 \times 2 \times 5 \times 5$, 2. (a) 18 (b) 24 (c) 30 (d) 75 (e) 105 (f) 45

☺ **Practice #2**

1. Find the unique set of prime factors for the following numbers.

- | | | | | |
|--------|--------|---------|---------|----------|
| (a) 4 | (d) 18 | (g) 46 | (j) 363 | (m) 1024 |
| (b) 10 | (e) 25 | (h) 88 | (k) 426 | (n) 1143 |
| (c) 16 | (f) 33 | (i) 130 | (l) 666 | (o) 1396 |

2. Find the prime factors of the following numbers.

- | | | | | |
|---------|----------|----------|---------|---------|
| (a) 6 | (e) 90 | (i) 2401 | (m) 144 | (p) 153 |
| (b) 66 | (f) 1155 | (j) 110 | (n) 385 | (q) 189 |
| (c) 625 | (g) 48 | (k) 357 | (o) 333 | (r) 201 |
| (d) 28 | (h) 630 | (l) 285 | | |

3. Find the prime factors of the following numbers.

- | | | | | |
|--------|---------|---------|----------|-----------|
| (a) 45 | (d) 87 | (g) 315 | (j) 891 | (m) 6567 |
| (b) 68 | (e) 168 | (h) 429 | (k) 1089 | (n) 14157 |
| (c) 72 | (f) 252 | (i) 756 | (l) 2751 | (o) 71995 |

Answer: 1. (a) $4=2 \times 2$ (b) $10=2 \times 5$ (c) $16=2 \times 2 \times 2 \times 2$ (d) $18=2 \times 3 \times 3$ (e) $25=5 \times 5$ (f) $33=3 \times 11$ (g) $46=2 \times 23$ (h) $88=2 \times 2 \times 2 \times 11$ (i) $130=2 \times 5 \times 13$ (j) $363=3 \times 11 \times 11$ (k) $426=2 \times 3 \times 71$ (l) $666=2 \times 3 \times 3 \times 37$ (m) $1024=2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ (n) $1143=3 \times 3 \times 41$ (o) $1396=2 \times 2 \times 349$ 2. (a) $6=2 \times 3$ (b) $66=2 \times 3 \times 11$ (c) $625=5 \times 5 \times 5 \times 5$ (d) $28=2 \times 2 \times 7$ (e) $90=2 \times 3 \times 3 \times 5$ (f) $1155=3 \times 5 \times 7 \times 11$ (g) $48=2 \times 2 \times 2 \times 2 \times 3$ (h) $630=2 \times 3 \times 3 \times 5 \times 7$ (i) $2401=7 \times 7 \times 7 \times 7$ (j) $110=2 \times 5 \times 11$ (k) $357=3 \times 7 \times 17$ (l) $285=3 \times 5 \times 19$ (m) $144=2 \times 2 \times 2 \times 2 \times 3 \times 3$ (n) $385=5 \times 7 \times 11$ (o) $333=3 \times 3 \times 37$ (p) $153=3 \times 3 \times 17$ (q) $189=3 \times 3 \times 3 \times 7$ (r) $201=3 \times 67$ 3. (a) $45=3 \times 3 \times 5$ (b) $87=3 \times 29$ (c) $72=2 \times 2 \times 2 \times 2 \times 3 \times 3$ (d) $87=3 \times 29$ (e) $168=2 \times 2 \times 2 \times 3 \times 7$ (f) $252=2 \times 2 \times 3 \times 3 \times 7$ (g) $315=3 \times 3 \times 5 \times 7$ (h) $429=3 \times 11 \times 13$ (i) $756=2 \times 2 \times 3 \times 3 \times 3 \times 7$ (j) $891=3 \times 3 \times 3 \times 3 \times 11$ (k) $1089=3 \times 3 \times 11 \times 11$ (l) $2751=3 \times 7 \times 131$ (m) $6567=3 \times 11 \times 199$ (n) $14157=3 \times 3 \times 11 \times 11 \times 13$ (o) $71995=5 \times 7 \times 11 \times 11 \times 17$

☺ **Practice #3**

1. Find the prime factors of the following numbers.

- | | | | | |
|---------|---------|----------|-----------|-------------|
| (a) 45 | (f) 252 | (k) 756 | (p) 6567 | (u) 333333 |
| (b) 56 | (g) 315 | (l) 891 | (q) 9768 | (v) 405769 |
| (c) 72 | (h) 429 | (m) 1089 | (r) 14157 | (w) 537152 |
| (d) 87 | (i) 512 | (n) 2751 | (s) 71996 | (x) 5666661 |
| (e) 168 | (j) 626 | (o) 4620 | (t) 89712 | (y) 5056506 |

2. Call out random numbers to the student and have him determine the prime factors. Do this until he or she can confidently factor a number with certainty.

$$\begin{aligned}
 16800 \div 5600 &= \frac{16800}{5600} && \text{(Factor out 100)} \\
 &= \frac{168}{56} \frac{21}{7} && \text{(Factor out 8)} \\
 &= \frac{21}{7} \frac{3}{1} && \text{(Factor out 7)} \\
 &= 3
 \end{aligned}$$

3. Division by factoring allows division before multiplication in mixed operations. This simplifies calculations considerably.

$$\begin{aligned}
 56 \times 9 \div 6 &= \frac{56 \times 9}{6} \\
 &= \frac{28}{6} \frac{56 \times 9}{3} && \text{(Factor out 2)} \\
 &= \frac{28}{6} \frac{56 \times 9^3}{3} && \text{(Factor out 3)} \\
 &= 28 \times 3 \\
 &= 84
 \end{aligned}$$

☺ Exercise B3.6

1. Divide by taking out the common factors

- | | | |
|--------------|--------------|------------------|
| (a) 36 ÷ 12 | (g) 189 ÷ 21 | (m) 806 ÷ 26 |
| (b) 98 ÷ 14 | (h) 350 ÷ 14 | (n) 966 ÷ 42 |
| (c) 125 ÷ 25 | (i) 272 ÷ 16 | (o) 3885 ÷ 105 |
| (d) 504 ÷ 36 | (j) 640 ÷ 40 | (p) 7920 ÷ 240 |
| (e) 980 ÷ 28 | (k) 783 ÷ 27 | (q) 60000 ÷ 2400 |
| (f) 270 ÷ 18 | (l) 544 ÷ 32 | (r) 17640 ÷ 630 |

Answer: 1. (a) 3 (b) 7 (c) 5 (d) 14 (e) 35 (f) 15 (g) 9 (h) 25 (i) 17 (j) 16 (k) 29 (l) 17 (m) 31 (n) 23 (o) 37 (p) 25 (q) 25 (r) 28

Lesson B3.7 Greatest Common Factor

The GREATEST COMMON FACTOR (GCF) is the largest factor that is shared by two or more numbers.

1. A factor that is common to two or more numbers is called a COMMON FACTOR.

- (a) 3 can be factored out of 18 and 12. Therefore, 3 is a common factor.

$$\frac{18}{12} = \frac{18}{12} \frac{6}{4} \quad \text{(Factor out 3)}$$

(b) 2 is a common factor of all even numbers.

(c) 7 is a common factor of all its multiples: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70 etc.

2. The largest factor that is common to two or more numbers is called the GREATEST COMMON FACTOR.

(a) The largest number that can be factored out of 18 and 12 is 6, because the remaining factors 3 and 2 have no factors in common.

$$\frac{18}{12} = \frac{\cancel{18}^3}{\cancel{12}_2} \quad (\text{Factor out } 6)$$

(b) The largest number that can be factored out of 28 and 42 is 14. Therefore, the GCF of 28 and 42 is 14.

(c) The largest number that can be factored out of 25 and 75 is 25. Therefore, the GCF of 25 and 75 is 25.

3. The GCF of two or more numbers is the product of all prime factors that can be factored out of those numbers.

(a) The prime factors of 45 and 60 are as follows:

$$\begin{aligned} 45 &= 3 \times 3 \times 5 \\ 60 &= 2 \times 2 \times 3 \times 5 \end{aligned}$$

The common prime factors are 3 and 5.

Therefore,

$$\text{GCF} = 3 \times 5 = 15$$

(b) We may find the common factors by short division more conveniently.

Divide the given numbers side by side by prime factors.

No more common factors can be taken out from 3 and 4.

The common prime factors are 3 and 5.

Therefore,

$$\text{GCF} = 3 \times 5 = 15$$

$$\begin{array}{r|l} 3 & \underline{45, 60} \\ 5 & \underline{15, 20} \\ & 3, 4 \end{array}$$

(c) Determine the GCF of 42, 56, and 70.

Divide the given numbers side by side by prime factors.

No more common factors can be taken out from 3, 4 and 5.

The common prime factors are 2 and 7.

Therefore,

$$\text{GCF} = 2 \times 7 = 14$$

$$\begin{array}{r|l} 2 & \underline{42, 56, 70} \\ 7 & \underline{21, 28, 35} \\ & 3, 4, 5 \end{array}$$

(d) Determine the GCF of 162, 729 and 4374.

Divide the given numbers side by side by prime factors.

No more common factors can be taken out from 2, 9 and 54.

The common prime factors are 3, 3, 3, and 3.

$$\text{GCF} = 3 \times 3 \times 3 \times 3 = 81$$

$$\begin{array}{r|l} 3 & \underline{162, 729, 4374} \\ 3 & \underline{54, 243, 1458} \\ 3 & \underline{18, 81, 486} \\ 3 & \underline{6, 27, 162} \\ & 2, 9, 54 \end{array}$$

4. When the prime factors cannot readily be found, the following procedure may be used to determine the GCF

- (i) Divide the greater number by the smaller number, and get the remainder.
- (ii) Divide the "last divisor" by the "last remainder." Get the new remainder.
- (iii) Repeat the step above until there is no remainder.
- (iv) Then the last divisor will be the GCF of the original two numbers.

(a) Determine the GCF of 5767 and 7081.

$$\begin{aligned}7081 \div 5767 &\rightarrow \text{Remainder} = 1314, \\5767 \div 1314 &\rightarrow \text{Remainder} = 511, \\1314 \div 511 &\rightarrow \text{Remainder} = 292, \\511 \div 292 &\rightarrow \text{Remainder} = 219, \\292 \div 219 &\rightarrow \text{Remainder} = 73, \\219 \div 73 &\rightarrow \text{No remainder}\end{aligned}$$

$$\text{GCF} = 73$$

(b) Determine the GCF of 2109, 3219, and 4181.

First determine the GCF of 2109 and 3219.

$$\begin{aligned}3219 \div 2109 &\rightarrow \text{Remainder} = 1110, \\2109 \div 1110 &\rightarrow \text{Remainder} = 999, \\1110 \div 999 &\rightarrow \text{Remainder} = 111, \\999 \div 111 &\rightarrow \text{No remainder}\end{aligned}$$

$$\text{GCF} = 111$$

Now determine the GCF of 111 and 4181.

$$\begin{aligned}4181 \div 111 &\rightarrow \text{Remainder} = 74, \\111 \div 74 &\rightarrow \text{Remainder} = 37, \\74 \div 37 &\rightarrow \text{No remainder}\end{aligned}$$

$$\text{GCF} = 37$$

(c) Numbers prime to one another.

Determine the GCF of 24 and 49.

$$\begin{aligned}49 \div 24 &\rightarrow \text{Remainder} = 1, \\24 \div 1 &\rightarrow \text{No Remainder}\end{aligned}$$

The GCF of 49 and 24 is 1. But 1 is common to all numbers. We find that the numbers 49 and 24 have no factors in common.

$$\begin{aligned}24 &= 2 \times 2 \times 2 \times 3 \\49 &= 7 \times 7\end{aligned}$$

Such numbers are called **prime to one another**.

5. Word problems with GCF are as follows.

- (a) Suppose you need to put 20391 gallons of beer and 49287 gallons of wine into an exact number of barrels, all of the same size and as big as possible, without mixing the beer and wine together. How will you find the amount each barrel must hold?

The answer is the Greatest Common Factor (GCF) of the numbers 20391 and 49287.

☺ Exercise B3.7

- Find the GCF (Greatest Common Factor) of

(a) 4 and 6	(d) 12 and 18	(g) 18, 27, and 36
(b) 18 and 27	(e) 54 and 258	(h) 162, 729 and 4374
(c) 30 and 42	(f) 216 and 258	(i) 492, 744 and 1044
- Find the GCF (Greatest Common Factor) of

(a) 936 and 2925	(f) 492, 744 and 1044
(b) 9756 and 8496	(g) 1326, 3094 and 4420
(c) 10353 and 14877	(h) 2697, 3441 and 1271
(d) 66429 and 169037	(i) 128, 136, 256, 442 and 940
(e) 94248 and 105336	(j) 102, 612, 476, 816 and 428
- Suppose you need to put 20391 gallons of beer and 49287 gallons of wine into an exact number of barrels, all of the same size and as big as possible, without mixing the beer and wine together. How will you find the amount each barrel must hold?

Answer: 1. (a) 2 (b) 9 (c) 6 (d) 6 (e) 6 (f) 6 (g) 9 (h) 81 (i) 12 2. (a) 117 (b) 36 (c) 87 (d) 121 (e) 5544 (f) 12 (g) 442 (h) 31 (i) 2 (j) 2 3. Each barrel must hold 21 gallons

SUMMARY

The use of FACTORS and PRIME NUMBERS has declined in the world of calculators today. However, a conceptual understanding of these concepts leads to insights that calculators and computers cannot provide.

PRIME NUMBERS

2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	1009
59	139	233	337	439	557	653	769	883	1013

The factors are obtained from EXACT DIVISION. The divisor and the quotient are the factors of the dividend. When a number cannot be factored into a pair of smaller numbers then it is a prime number.

A composite number has a unique set of prime factors.

The following is a list of prime numbers to a thousand or so. You may find this list useful.

You may now attempt to find the next ten prime numbers after 1013.

DIAGNOSTIC TEST

11. Determine the following.

- | | | |
|---------------------------|--------------------------|--------------------------|
| (a) Is 4 a factor of 13? | (c) Is 7 a factor of 49? | (e) Is 6 a factor of 56? |
| (b) Is 13 a factor of 91? | (d) Is 9 a factor of 72? | (f) Is 8 a factor of 42? |

12. Write a pair of factors for the following numbers (there is more than one answer).

- | | | |
|--------------------|--------------------|--------------------|
| (a) 18 = ___ x ___ | (c) 32 = ___ x ___ | (e) 40 = ___ x ___ |
| (b) 33 = ___ x ___ | (d) 23 = ___ x ___ | (f) 75 = ___ x ___ |

13. Identify the PRIME numbers.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 21, 23, 31, 39, 41, 49, 53, 57, 67, 79, 87, 91

14. Rapidly identify the numbers divisible by 5.

32, 65, 44, 87, 40, 57, 78, 100, 174, 465, 999, 5390, 8883, 5555, 9999993

15. Rapidly identify numbers divisible by 3.

32, 63, 44, 87, 66, 57, 78, 100, 174, 465, 999, 5392, 8883, 5556, 9999993

16. Write all even prime numbers less than 100.

17. Write all prime numbers between 100 and 120.

18. Find the prime factors of the following numbers.

36, 88, 630, 1155, 2401

19. Divide by taking out the common factors

- | | | |
|-------------------|-------------------|----------------------|
| (a) $36 \div 12$ | (c) $806 \div 26$ | (e) $7920 \div 240$ |
| (b) $270 \div 18$ | (d) $544 \div 32$ | (f) $17640 \div 630$ |

20. Determine the Greatest Common Factor (GCF) of.

- | | | |
|---------------|-----------------|-----------------------|
| (a) 18 and 27 | (c) 54 and 258 | (e) 162, 729 and 4374 |
| (b) 30 and 42 | (d) 216 and 264 | (f) 492, 744 and 1044 |

Answer: 1. (a) No (b) Yes (c) Yes (d) Yes (e) No (f) No 2. (a) 3x6 (b) 3x11 (c) 8x4 (d) 23x1 (e) 4x10
 (f) 3x25 3. 2, 3, 5, 7, 11, 23, 31, 41, 53, 67, 79 4. 65, 40, 100, 465, 5390, 5555 5. 63, 87,
 57, 78, 174, 465, 999, 8883, 5556, 9999993 6. 2, 7, 101, 103, 107, 109, 113 8. 36=2x2x3x3
 3x3, 88=2x2x2x11, 630=2x3x3x5x7, 1155=3x5x7x11, 2401=7x7x7x7 9. (a) 3 (b) 15 (c) 31
 (d) 17 (e) 33 (f) 28 10. (a) 9 (b) 6 (c) 6 (d) 24 (e) 81 (f) 12

GLOSSARY

[For additional words refer to the glossaries at the end of earlier Milestones]

- Common Factor** A **common factor** is an exact divisor of two or more numbers. For example, 4 is a common factor of 36 and 48 because 4 divides into 36 and 48 exactly.
- Composite Number** A **composite number** has at least one factor other than 1 and itself. Therefore, a factor pair may be found for the number which is not 1 and the number. For example, 4 is composite number because $4 = 2 \times 2$.
- Divisible** A number is **divisible** by another number, when it can be exactly divided by that number. For example, 91 is divisible by 7 means “ $91 \div 7$ ” is exact. A number is divisible by its factors.
- Factor** A **factor** is an exact divisor of a number. For example, 7 is a factor of 28 because the division “ $28 \div 7$ ” is exact. The word FACTOR comes from a Latin word meaning “doer, maker, performer.”
- Factor Pair** All numbers may be expressed as the product of two factors, which make a pair called **factor pair**. For example, 3 and 4 make a factor pair for 12, because $12 = 3 \times 4$.
- GCF** See GREATEST COMMON FACTOR
- Greatest Common Factor** The **greatest common factor** is the greatest number that is an exact divisor of two or more numbers. For example, 12 is the greatest common factor of 36 and 48 because 12 is the greatest number that divides into 36 and 48 exactly.
- Prime Factors** A **prime factor** is a factor that cannot be factored further. Every number has a unique set of prime factors, which, when multiplied together, produce that number. For example, the set of prime factors for 24 is $2 \times 2 \times 2 \times 3$.
- Prime Number** A **prime number** has no factors other than 1 and itself. Therefore, 1 and the prime number is the only factor pair that a prime number has. For example, 13 is a prime number because the only factor pair it has is $13 = 1 \times 13$.
- Prime to a Number** A number is prime to another number when their greatest common factor is 1. In other words, the two numbers have no factors in common other than 1. For example, the numbers 10 and 21 are prime to each other.