MATH MILESTONE # B1

MIXED OPERATIONS

The word, **milestone,** means "a point at which a significant change occurs." A Math Milestone refers to a significant point in the understanding of mathematics.

To reach this milestone one should be able to compute mixed operations easily and accurately.

A calculator shall be required to check the answers arrived at through mental math.

Please consult the **Glossary** supplied with this Milestone for mathematical terms. Consult a regular dictionary at **www.dictionary.com** for general English words that one does not understand fully.

You may start with the Diagnostic Test on the next page to assess your proficiency on this milestone. Then continue with the lessons with special attention to those, which address the weak areas.

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DIAGNOSTIC TEST

1. Compute the following.
   (a) 5 – 3 – 12 + 20 (c) 8 – 5 + 3 – 5 (e) 6 + 3 – 7 – 5 + 9 – 2
   (b) 5 – 7 + 8 – 4 (d) 3 + 8 + 4 – 6 – 7 – 2 (f) 3 – 5 – 2 + 9 – 3 – 4 – 5 + 7

2. Indicate which computations are incorrect. Why?
   (a) 9 – 4 + 3 – 1 = 9 – 7 – 1 = 2 – 1 = 1
   (b) 9 – 4 + 3 – 1 = 5 + 3 – 1 = 8 – 1 = 7
   (c) 13 – 6 – 4 + 3 = 7 – 4 + 3 = 3 + 3 = 6
   (d) 13 – 6 – 4 + 3 = 13 – 6 – 7 = 13 – 1 = 12

3. Compute the following.
   (a) 7 x 2 + 3 (c) 4 + 2 x 3 + 5 (e) 7 + (2 + 3) x (4 + 6) + 8
   (b) 7 + 2 x 3 (d) 6 x 3 + 4 (f) 7 x (6 – 2) – (8 – 4) x 6

4. Use parentheses to clarify the natural terms, and then solve.
   (a) 8 + 2 x 4 – 3 x 3 + 5 (c) 4 – 2 x 8 + 3 x 5 - 3 (e) 9 + 2 x 7 – 3 x 3 – 5
   (b) 8 x 2 – 4 x 3 + 3 x 5 (d) 6 x 2 – 4 x 3 – 3 + 7 (f) 7 x 6 – 2 x 5 – 4 x 6

5. Compute the following after rearranging as necessary.
   (a) 24 ÷ 6 x 4 ÷ 2 (c) 5 x 6 ÷ 3 ÷ 2 (e) 16 ÷ 7 x 21 ÷ 4
   (b) 5 x 16 ÷ 8 x 2 (d) 8 ÷ 5 x 20 ÷ 4 (f) 13 ÷ 2 ÷ 5 ÷ 13 x 10

6. Indicate which computations are incorrect. Why?
   (a) 4 x 9 ÷ 3 x 2 = 36 ÷ 6 = 6
   (b) 8 ÷ 4 x 2 ÷ 1 = 2 x 2 ÷ 1 = 4 ÷ 1 = 4
   (c) 30 ÷ 3 ÷ 5 x 2 = 10 ÷ 5 x 2 = 2 x 2 = 4
   (d) 30 ÷ 3 ÷ 5 x 2 = (30 ÷ 3) ÷ 5 x 2 = 10 ÷ 10 = 1

7. Solve the following after writing them in fraction form. Show the cancellations.
   (a) 6 x 16 x 5 ÷ 5 ÷ 6 ÷ 8 (d) 8 x 23 x 15 ÷ 5 ÷ 23 ÷ 8
   (b) 21 ÷ 8 x 2 ÷ 21 x 8 (e) 17 ÷ 8 x 5 ÷ 17 x 8
   (c) 13 ÷ 2 ÷ 5 ÷ 13 x 10 (f) 24 ÷ 8 ÷ 2 ÷ 24 x 32

8. Identify how many terms there are in each expression. You may put parentheses around the terms to clarify it.
   (a) 6 x 16 x 5 ÷ 5 ÷ 6 ÷ 8 (d) 3 + 5 x 4 – 8 ÷ 4 x 3 + 7
   (b) 21 ÷ 8 + 2 ÷ 21 – 8 (e) 5 x 9 ÷ 3 + 32 ÷ 2 ÷ 2 ÷ 2
   (c) 13 – 2 + 5 – 13 + 10 (f) 36 – 6 – 6 – 6 – 6 – 6 – 6

9. Reduce the following expressions to a number.
   (a) 6 x 6 – 5 x 6 + 3 ÷ 3 + 3 ÷ 3 – 4 (d) 3 + 5 x 4 – 8 ÷ 4 x 3 + 7 – 12 ÷ 3
   (b) 18 ÷ 9 + 55 ÷ 11 – 21 ÷ 3 + 2 + 1 (e) 21 ÷ 3 – 21 ÷ 7 + 8 x 3 ÷ 12 + 13
   (c) 8 ÷ 4 x 3 – 4 x 4 ÷ 2 + 6 ÷ 15 x 5 (f) 13 – 2 x 5 + 13 + 10 x 24 ÷ 16 + 5

Answer: **1.** (a) 10 (b) 2 (c) 1 (d) 0 (e) 4 (f) 0 **2.** (a) incorrect (b) correct (c) correct (d) incorrect **3.** (a) 17 (b) 13 (c) 15 (d) 22 (e) 65 (f) 4 **4.** (a) 12 (b) 19 (c) 0 (d) 4 (e) 9 (f) 8 **5.** (a) 8 (b) 20 (c) 5 (d) 20÷5 × 8÷4 = 4 × 2 = 8 (e) 16÷4 x 21÷7 = 4 x 3 = 12 (f) 13÷13 x 10÷2 ÷ 5 = 1 x 5÷5 =1x1 = 1 **6.** (a) Incorrect (b) Correct (c) Correct (d) Incorrect **7.** See Exercise B1.3, #7 **8.** (a) 1 term (b) 3 terms (c) 5 terms (d) 4 terms (e) 2 terms (f) 7 terms **9.** (a) 4 (b) 3 (c) 0 (d) 20 (e) 19 (f) 36

LESSONS

Lesson B1.1 Mixed Operations with + and –

Counting is the beginning of computation. Addition is "counting together," and therefore, it is an operation of first order. Subtraction, being opposite, or inverse, of addition, is also an operation of first order.

1. When addition and subtraction are present together they may be carried out from left to right in that sequence.

   $10 - 5 + 8 - 4 = 5 + 8 - 4 = 13 - 4 = 9$

   $3 + 9 - 4 + 7 = 12 - 4 + 7 = 8 + 7 = 15$

2. When we add 0 to the whole expression, the outcome remains the same; but we note that the "no sign" in front of the first number is actually a "plus" sign.

   $10 - 5 + 8 - 4 = 0 + 10 - 5 + 8 - 4$

3. The numbers may be moved around within the expression, but only if their operational signs are moved with them.

   $0 + 10 - 5 + 8 - 4 = 0 - 5 + 8 - 4 + 10 = 0 - 4 - 5 + 10 + 8$

4. We may total the minuends and subtrahends before subtraction.

   (a) $10 - 5 + 8 - 4 = (10 + 8) - (5 + 4) = 18 - 9 = 9$

   (b) $9 - 5 + 3 - 4 - 6 + 7 + 9 - 8 = (9 + 3 + 7 + 9) - (5 + 4 + 6 + 8)$
   $= 28 - 23$
   $= 5$

5. Numbers appearing both as minuend and subtrahend have the net affect of 0. We may, therefore, "cancel" them out in pairs.

   $8 - 5 + 3 - 4 - 3 + 7 + 4 - 8 = 8 - 5 + \cancel{3} - 4 - \cancel{3} + 7 + 4 - 8$
   $= 8 - 5 - \cancel{4} + 7 + \cancel{4} - 8$
   $= \cancel{8} - 5 + 7 - \cancel{8}$
   $= (7) - (5)$
   $= 2$

6. We may simplify minuend and subtrahend in pairs, where the minuend is larger.

   $10 - 5 + 8 - 4 = (10 - 5) + (8 - 4) = 5 + 4 = 9$

   $3 + 9 - 4 + 7 = 3 + (9 - 4) + 7 = 3 + 5 + 7 = 15$

   $5 + 9 - 4 - 7 = (5 - 4) + (9 - 7) = 1 + 2 = 3$

☺ Exercise B1.1

1. Compute the following.

   (a) 7– 4 + 2 (d) 9 – 3 – 4 + 1 (g) 23 – 18 + 7 – 12 + 8

   (b) 7 + 5 – 8 (e) 13 – 5 – 7 + 4 (h) 37 – 24 – 8 + 14 – 19

   (c) 2 + 6 – 5 (f) 15 – 9 + 6 – 5 (i) 43 + 21 – 60 +10 – 8

2. Rearrange the following mixed operations in at least three different ways.
   (a) 1 + 8 – 6 + 3 (b) 10 – 7 – 4 + 8 (c) 13 – 2 – 5 – 13 + 10

3. Indicate which computations are correct. Why?
   (e) 9 – 4 + 3 – 1 = 9 – 7 – 1 = 2 – 1 = 1
   (f) 9 – 4 + 3 – 1 = 5 + 3 – 1 = 8 – 1 = 7
   (g) 13 – 6 – 4 + 3 = 7 – 4 + 3 = 3 + 3 = 6
   (h) 13 – 6 – 4 + 3 = 13 – 6 – 7 = 13 – 1 = 12

4. Total the subtrahends before subtraction.
   (a) 19 – 6 – 3 – 5 = 19 – __ = ____
   (b) 23 – 7 – 9 – 2 = 23 – __ = ____
   (c) 16 – 9 – 3 – 2 = 16 – __ = ____

5. Total the minuends and subtrahends before subtraction.
   (a) 5 – 3 – 12 + 20 = ____ – ____ = ____
   (b) 5 – 3 + 8 – 7 = ____ – ____ = ____
   (c) 3 – 8 + 6 – 4 + 8 – 3 = ____ – ____ = ____

6. Cancel first to simplify the computation.
   (a) 6 + 7 + 5 – 1 – 6 – 8 (b) 7 – 6 + 2 – 7 + 6 (c) 10 – 6 – 5 – 10 + 5

7. Compute the following.
   (a) 12 – 7 – 8 – 2 + 8 (d) 9 – 5 – 9 + 5 + 2 (g) 6 + 7 – 16 + 3 – 5
   (b) 6 + 6 + 7 – 12 – 3 (e) 9 – 12 + 8 + 12 – 7 (h) 4 + 5 – 9 + 5 – 2
   (c) 16 + 7 – 16 + 3 – 5 (f) 3 – 6 + 7 – 9 + 3

8. Compute the following.
   (a) 15 – 3 – 10 + 2 (f) 6 – 2 – 4 + 3
   (b) 5 – 7 + 8 – 4 (g) 3 – 5 – 2 + 9 – 3 – 4 – 5 + 7
   (c) 3 + 8 + 4 – 6 – 7 – 2 (h) 6 + 3 – 7 – 5 + 9 – 2
   (d) 8 – 7 – 8 – 2 + 9 (i) 9 – 3 – 6 – 2 + 8 – 3 + 7 – 2
   (e) 8 + 8 – 16 + 5 - 2 (j) 7 – 6 – 5 – 11 + 5 + 9 – 2 + 7

Answer: **1.** (a) 5 (b) 4 (c) 3 (d) 3 (e) 5 (f) 7 (g) 8 (h) 0 (i) 6 **2.** (a) 8+3+1–6; 1–6+3+8; 3+1–6+8
(b) 8–7+10–4; 10–4+8–7; 8–4–7+10 (c) 13–13+10–2–5; 10+13–13–2–5; 10–5–2–13+13
3. (a) incorrect (b) correct (c) correct (d) incorrect **4.** (a) 19–14=5 (b) 23–18=5
(c) 16–14=2 **5.** (a) 25–15=10 (b) 13–10=3 (c) 17–15=2 **6.** (a) 12–9=3 (b) +2 (c) –6
7. (a) 12–9=3 (b) 7–3=4 (c) 10–5=5 (d) 2 (e) 17–7=10 (f) 13–15=–2 (g) –5 (h) 5–2=3
8. (a) 4 (b) 2 (c) 0 (d) 0 (e) 3 (f) 3 (g) 0 (h) 4 (i) 8 (j) 4

Lesson B1.2 Mixed Operations with x, + and –

Multiplication consists of repeated additions. Therefore, multiplication is an operation of second order. Multiplication takes priority over addition and subtraction in mixed operations.

Several additions		One multiplication
6 + 6 + 6 + 6 + 6	→	6 x 5
Lower Order Operation		Higher order Operation

1. In mixed operations, multiplication is carried out before addition.

$$7 + 2 \times 3 = 7 + (2 \times 3) = 7 + (2 + 2 + 2) = 7 + 6 = 13$$

Similarly,.

$$6 \times 3 + 2 = 18 + 2 = 20$$
$$2 + 3 \times 4 = 2 + 12 = 14$$

2. When subtraction is also present, the order of operations is first multiplication, then subtraction and addition.

$$3 \times 7 - 4 \times 5 + 3 = (3 \times 7) - (4 \times 5) + 3 = 21 - 20 + 3 = 1 + 3 = 4$$

$$7 \times 4 - 5 \times 3 + 5 = (7 \times 4) - (5 \times 3) + 5 = 28 - 15 + 5 = 13 + 5 = 18$$

3. Parentheses show that the expression within them is a single term. Multiplications are naturally part of a term. Terms are separated by + and −.

$$\begin{aligned} 2 + 3 \times 4 - 4 \times 7 - 3 + 6 \times 5 &= 2 + (3 \times 4) - (4 \times 7) - 3 + (6 \times 5) \\ &= 2 + 12 - 28 - 3 + 30 \\ &= 44 - 31 \\ &= 13 \end{aligned}$$

4. Any expression within the parentheses is resolved as a term. This could change the natural order of operations.

$$7 + 2 \times 3 = 7 + (2 \times 3) = 13 \quad \text{(natural order)}$$
$$(7 + 2) \times 3 = 9 \times 3 = 27 \quad \text{(changed order)}$$

$$6 \times 8 - 3 = (6 \times 8) - 3 = 45 \quad \text{(natural order)}$$
$$6 \times (8 - 3) = 6 \times 5 = 30 \quad \text{(changed order)}$$

5. We first reduce items within parentheses to single terms, and then follow the order of operations.
   (a) Reduce items within parentheses to single terms
   (b) Carry out the multiplications
   (c) Carry out subtractions and additions

$$\begin{aligned} 6 \times 8 - 3 \times (10 - 7) \times (7 - 2) &= 6 \times 8 - 3 \times 3 \times 5 \\ &= 48 - 45 \\ &= 3 \end{aligned}$$

$$\begin{aligned} (5 + 8) \times 2 - (4 + 3) \times 4 + 2 \times 8 &= 13 \times 2 - 7 \times 4 + 2 \times 8 \\ &= 26 - 28 + 16 \\ &= 14 \end{aligned}$$

☺ Exercise B1.2

1. Use parentheses to clarify the natural terms, and then solve.
   (a) 8 + 2 x 4 (d) 3 + 8 x 5 (g) 4 x 3 + 2 x 5
   (b) 8 x 2 + 4 (e) 8 x 3 + 5 (h) 4 + 3 x 2 + 5
   (c) 8 x 4 + 2 (f) 8 + 5 x 3 (i) 10 + 22 x 2

2. Use parentheses to clarify the natural terms, and then solve.

(a) 8 + 2 x 4 – 3 x 3 + 5	(c) 4 – 2 x 8 + 3 x 5 - 3	(e) 9 + 2 x 7 – 3 x 3 – 5
(b) 8 x 2 – 4 x 3 + 3 x 5	(d) 6 x 2 – 4 x 3 – 3 + 7	(f) 7 x 6 – 2 x 5 – 4 x 6

3. Solve.

(a) (8 + 2) x 4	(c) (4 + 2) x (8 – 4)	(e) 9 + 2 x (7 – 3) – (5 + 3)
(b) 7 x (9 – 6)	(d) 6 x 5 – 4 x (3 – 3 + 7)	(f) 7 x (6 – 2) – (8 – 4) x 6

4. Solve.

(a) 6 + 3 x 4	(f) 3 x 4 + 2 + 6 x 3 + 7 + 9
(b) (6 + 3) x 4	(g) (5 + 8) x 2 + (4 + 3) x 4 + 2 x 8
(c) 6 x 3 + 4	(h) 7 x 3 + 4 x 5 + 3 x 5
(d) (4 + 2) x (3 + 5)	(i) 7 + (7 + 4) x (5 + 6) + 8
(e) 4 + 2 x 3 + 5	(j) 7 x (5 + 3) + 5 x 8 + 6

Answer: **1.** (a) 16 (b) 20 (c) 34 (d) 43 (e) 29 (f) 23 (g) 22 (h) 15 (i) 54 **2.** (a) 12 (b) 19 (c) 0
(d) 4 (e) 9 (f) 8 **3.** (a) 40 (b) 21 (c) 24 (d) 2 (e) 9 (f) 4 **4.** (a) 18 (b) 36 (c) 22 (d) 48
(e) 15 (f) 48 (g) 70 (h) 56 (i) 136 (j) 102

Lesson B1.3 Mixed Operations with x and ÷

Division is opposite, or inverse, of multiplication. Therefore, Division is also an operation of the second order.

1. When multiplication and division are present together they may be carried out from left to right.

20 ÷ 5 x 8 ÷ 4 = 4 x 8 ÷ 4 = 32 ÷ 4 = 8
18 ÷ 6 x 14 ÷ 7 = 3 x 14 ÷ 7 = 42 ÷ 7 = 6

2. When we multiply the whole expression by 1, the outcome remains the same; but we note that the "no sign" in front of the first number is actually a "multiplication" sign.

18 ÷ 7 x 14 ÷ 9 = 1 x 18 ÷ 7 x 14 ÷ 9

3. The numbers may be moved around within the expression, but only if their operational signs are moved with them.

18 ÷ 7 x 14 ÷ 9 = 18 ÷ 9 x 14 ÷ 7 = 2 x 2 = 4

4. We may write the expression in fraction form by placing all the dividends above the line (as a product), and the divisors below the line (also as a product).

(a) $6 \times 8 \div 4 \div 3 = \frac{6 \times 8}{4 \times 3}$

(b) $40 \div 5 \times 8 \div 4 = \frac{40 \times 8}{5 \times 4}$

(c) $18 \div 7 \div 9 \times 14 = \frac{18 \times 14}{7 \times 9}$

5. We may match dividends with divisors and then carry out division first.

$$18 \div 7 \times 14 \div 9 = \frac{18 \times 14}{7 \times 9} = \frac{18 \times 14}{9 \times 7} = \frac{18}{9} \times \frac{14}{7} = 2 \times 2 = 4$$

6. Numbers appearing both as dividend and divisor have the net affect of 1. We may, therefore, "cancel" them out in pairs.

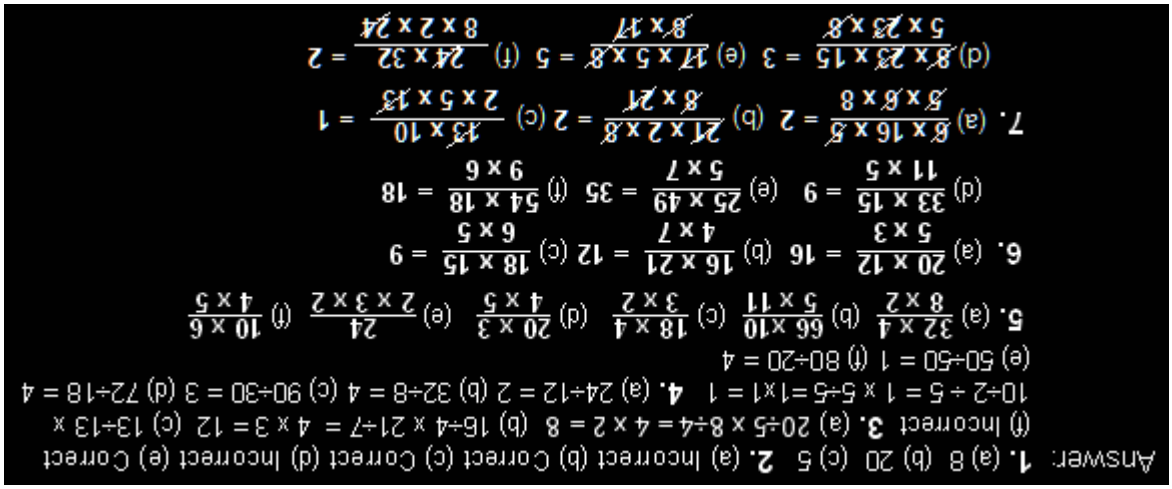
$$3 \div 5 \times 8 \div 4 \times 5 \div 3 = \frac{3 \times 8 \times 5}{5 \times 4 \times 3} = \frac{3 \times 8 \times 5}{3 \times 4 \times 5} = \frac{\cancel{3}^{1}}{\cancel{3}} \times \frac{8}{4} \times \frac{\cancel{5}^{1}}{\cancel{5}} = \frac{8}{4} = 2$$

$$8 \div 7 \times 5 \div 8 \times 7 = \frac{8 \times 5 \times 7}{7 \times 8} = \frac{\cancel{8} \times 5 \times \cancel{7}}{\cancel{8} \times \cancel{7}} = 5$$

☺ Exercise B1.3

1. Compute the following from left to right, or by division first.
   (a) 24 ÷ 6 x 4 ÷ 2 (b) 5 x 16 ÷ 8 x 2 (c) 5 x 6 ÷ 3 ÷ 2
2. Indicate which computations are incorrect and why?
   (e) 4 x 9 ÷ 3 x 2 = 36 ÷ 6 = 6
   (f) 4 x 9 ÷ 3 x 2 = 4 x (9 ÷ 3) x 2 = 4 x 3 x 2 = 24
   (g) 8 ÷ 4 x 2 ÷ 1 = 2 x 2 ÷ 1 = 4 ÷ 1 = 4
   (h) 8 ÷ 4 x 2 ÷ 1 = 8 ÷ 8 ÷ 1 = 1 ÷ 1 = 1
   (i) 30 ÷ 3 ÷ 5 x 2 = 10 ÷ 5 x 2 = 2 x 2 = 4
   (j) 30 ÷ 3 ÷ 5 x 2 = (30 ÷ 3) ÷ 5 x 2 = 10 ÷ 10 = 1
3. Rearrange for convenience and compute the following mixed operations.
   (a) 8 ÷ 5 x 20 ÷ 4 (b) 16 ÷ 7 x 21 ÷ 4 (c) 13 ÷ 2 ÷ 5 ÷ 13 x 10
4. Combine multiple divisors into one divisor.
   (a) 24 ÷ 2 ÷ 3 ÷ 2 (c) 90 ÷ 5 ÷ 3 ÷ 2 (e) 50 ÷ 5 ÷ 5 ÷ 2
   (b) 32 ÷ 2 ÷ 2 ÷ 2 (d) 72 ÷ 2 ÷ 3 ÷ 3 (f) 80 ÷ 2 ÷ 2 ÷ 5
5. Write the following mixed operations in "fraction" form.
   (a) 32 ÷ 8 x 4 ÷ 2 (c) 18 x 4 ÷ 3 ÷ 2 (e) 24 ÷ 2 ÷ 3 ÷ 2
   (b) 66 ÷ 5 x 10 ÷ 11 (d) 20 ÷ 4 ÷ 5 x 3 (f) 10 ÷ 4 ÷ 5 x 6
6. Solve the following after writing them in fraction form.
   (a) 20 ÷ 3 x 12 ÷ 5 (c) 18 ÷ 5 x 15 ÷ 6 (e) 25 ÷ 7 x 49 ÷ 5
   (b) 16 ÷ 7 x 21 ÷ 4 (d) 33 ÷ 5 x 15 ÷ 11 (f) 54 ÷ 6 x 18 ÷ 9

7. Solve the following after writing them in fraction form. Show the cancellations.
   (a) 6 x 16 x 5 ÷ 5 ÷ 6 ÷ 8
   (b) 21 ÷ 8 x 2 ÷ 21 x 8
   (c) 13 ÷ 2 ÷ 5 ÷ 13 x 10
   (d) 8 x 23 x 15 ÷ 5 ÷ 23 ÷ 8
   (e) 17 ÷ 8 x 5 ÷ 17 x 8
   (f) 24 ÷ 8 ÷ 2 ÷ 24 x 32



Lesson B1.4 Mixed Operations with x, ÷, + and –

Mixed operations are made up of <u>terms</u> containing Multiplications and Divisions. These terms are separated by Additions and subtractions. Terms are resolved first and then the rest.

1. Mathematical **terms** are made up of x and ÷. Such terms are separated by + and – in a mathematical expression.

   (a) The following expression is made up of 3 terms, which are separated by + and –, as pointed out by the arrows. Note that x and ÷ appear within the terms.

   8 x 3 + 4 ÷ 2 – 2 x 4
   term ↑ term ↑ term

   (b) We may clarify the terms by putting them within parentheses as follows.

   8 x 3 + 4 ÷ 2 – 2 x 4 → (8 x 3) + (4 ÷ 2) – (2 x 4)

   (c) A term may consist of a single number. For example, the following expression also consists of three terms.

   2 – 1 + 3 → (2) – (1) + (3)

   (d) If a mathematical expression has no + or –, then it consists of a single term. The following is a single term.

   12 x 7 ÷ 6 ÷ 4 x 2 x 18 ÷ 9 → (12 x 7 ÷ 6 ÷ 4 x 2 x 18 ÷ 9)

   (e) Here are some examples of terms in mathematical expressions. You may check out the number of terms by putting parenthesis around them.

2	**(1 term only)**
8 x 3 + 4 ÷ 2	**(2 terms)**
8 x 3 ÷ 12	**(1 term only)**
12 x 14 + 6 ÷ 4 – 2 x 18 + 9 + 5	**(5 terms)**

2 + 3	(2 terms)
12 x 14 ÷ 6 ÷ 4 + 2 x 18 ÷ 9	(2 terms)

2. In a mixed operation, terms are computed first, and then the whole expression.

   (a) Identify the terms by putting parentheses around them. Compute the terms first. Then compute the whole expression.

$$\begin{aligned} 8 \times 3 + 4 \div 2 - 2 \times 4 &= (8 \times 3) + (4 \div 2) - (2 \times 4) \\ &= 24 + 2 - 8 \\ &= 18 \end{aligned}$$

   (b) Here is another example.

$$\begin{aligned} 12 \times 14 \div 6 \div 4 - 2 \times 18 \div 9 + 5 & \\ &= (12 \times 14 \div 6 \div 4) - (2 \times 18 \div 9) + 5 \\ &= 7 - 4 + 5 \\ &= 8 \end{aligned}$$

☺ Exercise B1.4

1. Identify how many terms there are in each expression. You may put parentheses around the terms to clarify it.

   (d) 6 x 16 x 5 ÷ 5 ÷ 6 ÷ 8
   (e) 21 ÷ 8 + 2 ÷ 21 – 8
   (f) 13 – 2 + 5 – 13 + 10

   (d) 3 + 5 x 4 – 8 ÷ 4 x 3 + 7
   (e) 5 x 9 ÷ 3 + 32 ÷ 2 ÷ 2 ÷ 2
   (f) 36 – 6 – 6 – 6 – 6 – 6 – 6

2. Reduce the following expressions to a number.

   (d) 6 x 6 – 5 x 6 + 3 ÷ 3 + 3 ÷ 3 – 4
   (e) 18 ÷ 9 + 55 ÷ 11 – 21 ÷ 3 + 2 + 1
   (f) 8 ÷ 4 x 3 – 4 x 4 ÷ 2 + 6 ÷ 15 x 5

   (d) 3 + 5 x 4 – 8 ÷ 4 x 3 + 7 – 12 ÷ 3
   (e) 21 ÷ 3 – 21 ÷ 7 + 8 x 3 ÷ 12 + 13
   (f) 13 – 2 x 5 + 13 + 10 x 24 ÷ 16 + 5

3. Compute the following.

   (a) 8 x 3 ÷ 12 = _____
   (b) 13 x 7 ÷ 8 = _____
   (c) 20 x 3 ÷ 5 ÷ 6 = _____
   (d) 3 x 4 + 9 ÷ 3 = _____
   (e) 30 ÷ 4 ÷ 3 ÷ 5 = _____
   (f) 2 x 6 x 7 x 1 ÷ 14 = _____
   (g) 21 x 6 ÷ 7 ÷ 3 = _____
   (h) 5 x 6 ÷ 10 + 5 x 8 ÷ 20 = _____
   (i) 5 ÷ 6 x 5 x 2 ÷ 25 = _____
   (j) 8 x 6 + 12 x 5 ÷ 3 + 12 = _____
   (k) 21 x 35 x 24 ÷ 49 ÷ 15 ÷ 12 = _____
   (l) 128 ÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 = _____
   (m) 56 x 54 ÷ 6 x 5 ÷ 8 ÷ 9 = _____
   (n) 6 x 6 – 6 x 6 + 3 ÷ 3 – 3 ÷ 3 + 4 = _____
   (o) 18 ÷ 11 x 55 ÷ 14 x 21 ÷ 9 = _____
   (p) 8 ÷ 6 x 3 + 4 x 2 – 2 x 6 ÷ 4 = _____

Answer: **1.** (a) 1 term (b) 3 terms (c) 5 terms (d) 4 terms (e) 2 terms (f) 7 terms **2.** (a) 4 (b) 3 (c) 0 (d) 20 (e) 19 (f) 36 **3.** (a) 2 (b) 11 and 3/8 (c) 2 (d) 15 (e) 1/2 (f) 6 (g) 6 (h) 5 (i) 1/3 (j) 80 (k) 2 (l) 1 (m) 35 (n) 4 (o) 15 (p) 9

SUMMARY

When the various operations of addition, subtraction, multiplication and division are present together in an **arithmetic expression** we have **mixed operations**.

Counting is the beginning of computation. Addition is “counting together,” and therefore, it is an operation of **first order**. Subtraction, being opposite, or inverse, of addition, is also an operation of first order. When addition and subtraction are present together they may be carried out from left to right in that sequence. An operation on the right may be carried out first only when there is addition on its left.

Multiplication consists of repeated additions. Therefore, multiplication is an operation of **second order**. Division, being opposite, or inverse, of multiplication is also an operation of second order. When multiplication and division are present together they may be carried out from left to right in that sequence.

In mixed operations, **second order** operations take priority over **first order** operations.

Multiplication and division, when grouped together, are referred to as **terms**. Numbers and operations within parentheses are also called terms. The terms are separated from each other by plus or minus.

Terms are always reduced first before reducing rest of the arithmetic expression. Thus, the concept of terms automatically enforces the precedence of second order operations over first order operations.

Understanding the logic involved in reducing mixed operations is very important. Only when this logic is understood, do the “formulas” about precedence of operations make sense.

DIAGNOSTIC TEST

10. Compute the following.

(a) 5 – 3 – 12 + 20
(b) 5 – 7 + 8 – 4
(c) 8 – 5 + 3 – 5
(d) 3 + 8 + 4 – 6 – 7 – 2
(e) 6 + 3 – 7 – 5 + 9 – 2
(f) 3 – 5 – 2 + 9 – 3 – 4 – 5 + 7

11. Indicate which computations are incorrect. Why?

(i) 9 – 4 + 3 – 1 = 9 – 7 – 1 = 2 – 1 = 1
(j) 9 – 4 + 3 – 1 = 5 + 3 – 1 = 8 – 1 = 7
(k) 13 – 6 – 4 + 3 = 7 – 4 + 3 = 3 + 3 = 6
(l) 13 – 6 – 4 + 3 = 13 – 6 – 7 = 13 – 1 = 12

12. Compute the following.

(c) 7 x 2 + 3
(d) 7 + 2 x 3
(c) 4 + 2 x 3 + 5
(d) 6 x 3 + 4
(e) 7 + (2 + 3) x (4 + 6) + 8
(f) 7 x (6 – 2) – (8 – 4) x 6

13. Use parentheses to clarify the natural terms, and then solve.

(c) 8 + 2 x 4 – 3 x 3 + 5	(c) 4 – 2 x 8 + 3 x 5 - 3	(e) 9 + 2 x 7 – 3 x 3 – 5
(d) 8 x 2 – 4 x 3 + 3 x 5	(d) 6 x 2 – 4 x 3 – 3 + 7	(f) 7 x 6 – 2 x 5 – 4 x 6

14. Compute the following after rearranging as necessary.

(a) 24 ÷ 6 x 4 ÷ 2	(c) 5 x 6 ÷ 3 ÷ 2	(e) 16 ÷ 7 x 21 ÷ 4
(b) 5 x 16 ÷ 8 x 2	(d) 8 ÷ 5 x 20 ÷ 4	(f) 13 ÷ 2 ÷ 5 ÷ 13 x 10

15. Indicate which computations are incorrect. Why?

(k) 4 x 9 ÷ 3 x 2	=	36 ÷ 6	=	6		
(l) 8 ÷ 4 x 2 ÷ 1	=	2 x 2 ÷ 1	=	4 ÷ 1	=	4
(m) 30 ÷ 3 ÷ 5 x 2	=	10 ÷ 5 x 2	=	2 x 2	=	4
(n) 30 ÷ 3 ÷ 5 x 2	=	(30 ÷ 3) ÷ 5 x 2	=	10 ÷ 10	=	1

16. Solve the following after writing them in fraction form. Show the cancellations.

(d) 6 x 16 x 5 ÷ 5 ÷ 6 ÷ 8	(d) 8 x 23 x 15 ÷ 5 ÷ 23 ÷ 8
(e) 21 ÷ 8 x 2 ÷ 21 x 8	(e) 17 ÷ 8 x 5 ÷ 17 x 8
(f) 13 ÷ 2 ÷ 5 ÷ 13 x 10	(f) 24 ÷ 8 ÷ 2 ÷ 24 x 32

17. Identify how many terms there are in each expression. You may put parentheses around the terms to clarify it.

(g) 6 x 16 x 5 ÷ 5 ÷ 6 ÷ 8	(d) 3 + 5 x 4 – 8 ÷ 4 x 3 + 7
(h) 21 ÷ 8 + 2 ÷ 21 – 8	(e) 5 x 9 ÷ 3 + 32 ÷ 2 ÷ 2 ÷ 2
(i) 13 – 2 + 5 – 13 + 10	(f) 36 – 6 – 6 – 6 – 6 – 6 – 6

18. Reduce the following expressions to a number.

(g) 6 x 6 – 5 x 6 + 3 ÷ 3 + 3 ÷ 3 – 4	(d) 3 + 5 x 4 – 8 ÷ 4 x 3 + 7 – 12 ÷ 3
(h) 18 ÷ 9 + 55 ÷ 11 – 21 ÷ 3 + 2 + 1	(e) 21 ÷ 3 – 21 ÷ 7 + 8 x 3 ÷ 12 + 13
(i) 8 ÷ 4 x 3 – 4 x 4 ÷ 2 + 6 ÷ 15 x 5	(f) 13 – 2 x 5 + 13 + 10 x 24 ÷ 16 + 5

Answer: **1.** (a) 10 (b) 2 (c) 1 (d) 0 (e) 4 (f) 0 **2.** (a) incorrect (b) correct (c) correct (d) incorrect
3. (a) 17 (b) 13 (c) 15 (d) 22 (e) 65 (f) 4 **4.** (a) 12 (b) 19 (c) 0 (d) 4 (e) 9 (f) 8 **5.** (a) 8
(b) 20 (c) 5 (d) 20÷5 × 8÷4 = 4 × 2 = 8 (e) 16÷4 x 21÷7 = 4 x 3 = 12 (f) 13÷13 x 10÷2 ÷ 5
= 1 x 5÷5 =1x1 = 1 **6.** (a) Incorrect (b) Correct (c) Correct (d) Incorrect **7.** See Exercise
B1.3, #7 **8.** (a) 1 term (b) 3 terms (c) 5 terms (d) 4 terms (e) 2 terms (f) 7 terms
9. (a) 4 (b) 3 (c) 0 (d) 20 (e) 19 (f) 36

GLOSSARY

[For additional words refer to the glossaries at the end of earlier Milestones]

Expression — A **mathematical expression** describes operations on numbers. For example, "3 x 4 – 2 + 12 ÷ 6" is a mathematical expression that reduces to a numeric value.

Mixed Operation: — When more than one operation (addition, subtraction, multiplication and division) is present in a mathematical expression, we have **mixed operations.**

Term — A **term** is that part of a mathematical expression that consists entirely of multiplication and division. Multiple terms in an expression are separated by addition and subtraction.